Have there been differences between the growth rates in different periods of the development of the capitalist world economy since 1850? An application of cluster analysis in time series analysis

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Cluster analysis is a method that can be employed to classify objects with different features in such a way that very similar objects are combined in one class (cluster), whereas dissimilar objects are combined in other classes (clusters). From a mathematical point of view, the objects are points in a multidimensional space, and the dimension of the space is given by the number of features. In pattern recognition terminology, the points are called patterns or OTU's (operational taxonomic units), the axes of the pattern space are the features or measurements.

Many different methods of cluster analysis are now in use, and the results of alternative methods are often not comparable. Hence the majority of mathematicans regard cluster analysis as a subjective technique, the application and interpretation of which depend on the user's standpoint, experience, and perspicacity. Nevertheless we hope to suggest an easy procedure for clustering time series that is not only usable but is also interpretable in a more objective way.

We can formulate the aim of the application of cluster analysis in terms of two requirements:

1. The variations within clusters should be as slight as possible.
2. The variations between clusters should be as large as possible.

For a mathematical formulation of the problem we introduce the following symbols and definitions:

$t = 1, 2, \ldots, N$ – index of objects
$i = 1, 2, \ldots, m$ – index of clusters
$j = 1, 2, \ldots, n_i$ – index of objects within the i–th cluster
$k = 1, 2, \ldots, p$ – index of features
x_{ijk} – k–th feature of the j–th element within the i–th cluster
$x_{ik} = \sum x_{ijk}/n_i$ – k–th feature of the i–th cluster-mean
$x_k = \sum x_{ik}/m$ – (unweighted) mean of the cluster-means of the k–th feature

We now have to answer the question of how to measure the variations. The most well-known formulation for the measurement of variations is the variance, and

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many cluster methods are based on the computation of within- and between-cluster-variances. But we must not forget that in this way we would compute the measure in squared units. For instance the objective function of a cluster analysis of population data would be formulated in „squared persons", and that is – from every non-mathematical point of view – nonsense. A second possibility of variation measurement is the square root of the variance, the standard deviation. The disadvantage of the standard deviation is that it depends upon magnitude of the features. In respect to different features this disadvantage could be eliminated by standardization. But in this way the problem is not soluble in respect to one feature, since the absolute difference between two units of one feature still depends upon their size. Larger values, mostly, involve larger differences, and vice versa. Therefore, we should use the coefficient of variation (V).

$$V(x_{ik}) = \sqrt{\frac{1}{n_i - 1} \sum_j (x_{ijk}/x_{ik} - 1)^2} \tag{1}$$

The within-cluster variation in respect to the k-th feature is measurable as the weighted mean of the $V(x_{ik})$, i. e.

$$V_k^{(w)} = \frac{1}{N} \sum_i n_i V(x_{ik}) \tag{2}$$

Analogous to formula (1), we compute the between-cluster variation:

$$V_k^{(b)} = \sqrt{\frac{1}{m-1} \sum_i (x_{ik}/x_k - 1)^2} \tag{3}$$

The two requirements indicated above can be formulated for each feature as extremum problems:

$$V_k^{(w)} \overset{!}{=} \min \tag{4a}$$

$$V_k^{(b)} \overset{!}{=} \max \tag{4b}$$

Of course, the difference $V_k^{(b)} - V_k^{(w)}$ is the objective function that must be maximized for each feature:

$$c_k = V_k^{(b)} - V_k^{(w)} \overset{!}{=} \max \tag{5a}$$

Consequently, the objective function of the whole system is:

$$C = \sum_k g_k c_k \overset{!}{=} \max \quad (\text{with} \sum_k g_k = 1) \tag{5b}$$

where g_k are the weights of the features (if we compute C without weights then each g_k equals $1/p$).

In general in cluster analysis, the optima of the objective functions C are not computable because the number of possible solutions is $(N+1)^N$, and no computer has a memory large enough to solve the problem. Hence we must introduce restrictions and/or a priori informations, e. g. a given number of clusters or a given minimum of within-cluster variation, or we used so called heuristic procedures, and in that case we again land at the starting point and come face to face with „the user's dilemma"[1].

But I consider that all those heuristic procedures, restrictions, a priori information etc. are not necessary in order to carry out a time series analysis by means of cluster analysis. Firstly we do not use a method based on a minimization of squared errors – like FORGY, ISODATE, WISH etc. – that would force us to give a minimum[2]. Secondly, a time series is a well-ordered set, the sequence of data is given – even as a time series. The problem whether or not the t-th and the (t+v)-th year are elements of the same, let us say the i-th period, is soluble by means of stepwise clustering. If we state for the (t+u)-th year ($u < v$) that is not an element of the i-th period, then, of course, neither is the (t+v)-th year an element of that period. Hence, and thirdly, we are not forced to specify the number of clusters (like in CLUSTER). We can use an agglomerative method of cluster analysis for the *mathematical* solution of periodization problems.

The criterion for this periodization is a purely mathematical one, i. e. the subjective option of both the historian and the mathematician is eliminated, provided, howefer, that we accept this criterion as such. Acceptance of the criterion however, depends on whether or not it reflects the mathematical structure of the object under investigation. But we can only answer this question from a historical point of view, the basis for the applicability of a mathematical method is to be found in the object under investigation, and is not to be found in the mathematical method itself. It is absolutely necessary to stress this fact because the application of mathematical methods in any field of science without a scientific that is a non-mathematical foundation is only nonsense. Quite rightly, Hegel spoke about the mathematical conclusion as the exterior conclusion („äußerer Schluß")[3].

Moreover, clustering techniques are tools for discovery rather than ends in themselves and should permit the user form statistical questions for further studies[4]. In the case of time series analysis, one such question would concern the significance of the difference between the within-periods-means.

As an example we analyze the growth of production and trade (in terms of constant prices) in the capitalist world since 1850. Figures 1–7 show us the growth of

[1] Dubes, R., and Jain, A. K., Clustering Techniques: The User's Dilemma, in: Pattern Recognition, 8 (1976), pp. 247 ff.

[2] Otherwise we would only obtain the trivial solution: Only years with identical features would constitute a cluster.

[3] Hegel, Encyclopedia of Philosophical Science (1830), § 188 (v. Wissenschaft der Logik, Vol. III, ch. 1.3.A.d.).

[4] Dubes, Techniques.

- industrial production
- agricultural production
- total production (incl. mining)
- total exports

in Million Dollars

- ratio of industrial production
- ratio of agricultural production
- ratio of total exports

to total production in promille

By means of regression analysis we can demonstrate that the growth of production and trade over the past 125 years was exponential[5]. The quotient of two exponentially growing indicators also grows exponentially. Consequently, we can say that the capitalist world economy is an expanding economy. Therefore it is useless to cluster the series of states (as given in Appendix I)[6], for the result of such an attempt is a very large number of clusters with a very small number of elements, even for years with similar states. For example the result of such a cluster analysis of the industrial production of states is division of this time series into 33 clusters. This fact only shows that the mathematical structure of this time series is not such that we can use the method in question. The result of the application of the method shows that the method is not applicable.

Because of the exponential growth of the capitalist world economy, we compute a time series of growth rates; symbolizing the quanta produced or sold by x_t, we symbolize the indices by

$$y_t = 100\, x_t/x_{t-1} \tag{6}$$

Then the theoretical values are

$$\hat{x} = ab^t \tag{7a}$$

$$\hat{y} = 100\, \hat{x}_t/\hat{x}_{t-1} = 100 ab^t/ab^{t-1} = 100\, b \tag{7b}$$

The empirical indices y_t oscillate around their theoretical values 100 b. Therefore it is useful to analyze indices. As to our mathematical apparatus, it would be a mistake to apply it to the indices y_t because the method of least squares cannot be employed for the analysis of quotients. This follows from the fact that the arithmetic mean of quotients does not equal the quotient of arithmetic means:

$$\frac{100}{n} \sum x_t/x_{t-1} \neq \frac{\frac{100}{n} \sum x_t}{\frac{1}{n} \sum x_{t-1}} \tag{8a}$$

[5] Kuczynski, Th., Spectral Analysis and Cluster Analysis as Mathematical Methods for the Periodization of Historical Processes – a Comparison of Results Based on Data about the Development of Production and Innovation in the History of Capitalism. Kondratieff Cycles – Appearance or Reality, in: Proceedings of the Seventh International Economic History Congress, Edinburgh 1978, pp. 79.

[6] Sources: Kuczynski, J., Die Geschichte der Lage der Arbeiter unter dem Kapitalismus, Vol. 37, Berlin 1967, p. 31, 78 (1850–1964); UNO-Monthly Bulletin of Statistics (1964–1976).

Therefore we use the logarithmic indices

$$z_t = 2 + \lg x_t - \lg x_{t-1} \tag{9a}$$

which oscillate around the expected value

$$\hat{z}_t = 2 + \lg b \tag{9b}$$

The arithmetic mean of the logarithmic indices $M_a(z)$ equals the logarithm of the geometric mean of the indices $\lg(M_g(y))$:

$$\begin{aligned} M_a(z) &= \frac{1}{n} \sum (2 + \lg x_t - \lg x_{t-1}) \\ &= 2 + \frac{1}{n} (\lg x_n - \lg x_o) \\ &= \lg(100 \sqrt[n]{x_n/x_o}) \\ &= \lg(\sqrt[n]{\prod 100 x_t/x_{t-1}}) \end{aligned} \tag{10a}$$

hence

$$M_a(\lg y) = \lg(M_g(y)) \tag{10b}$$

Table 1 shows the logarithmic growth rates
(1) agricultural production (A)
(2) industrial production (I)
(3) total production (including mining) (T)
(4) total exports (E)
(5) the ratio of agricultural to total production (a)
(6) the ratio of industrial to total production (i)
(7) the ratio of total exports to total production (e)

We can now ask a more correct question: Are there differences between the logarithmetic indices in different periods of development of capitalist world economy since 1850? We want to try to answer this question by using the clustering technique described above.

If we cluster the six-dimensional time series (without (4)) the optimum result is given by a division of these series into two clusters: 1851–1945, 1946–1976. We get a local optimum if we construct the clusters 1850–1920, 1921, 1922, 1923–1942, 1943, 1944, 1945, 1946, 1947–1976. The results of the one-dimensional cluster analysis of all seven time series are similar. Because of the world wars, which most strongly affected the (logarithmic) indices, the results are almost useless.

Because there is such great irregularity in the indices between the world wars, we build an a priori cluster and repeat the cluster analysis. The results are completely different. Before World War I, we can differentiate three periods in all seven series;

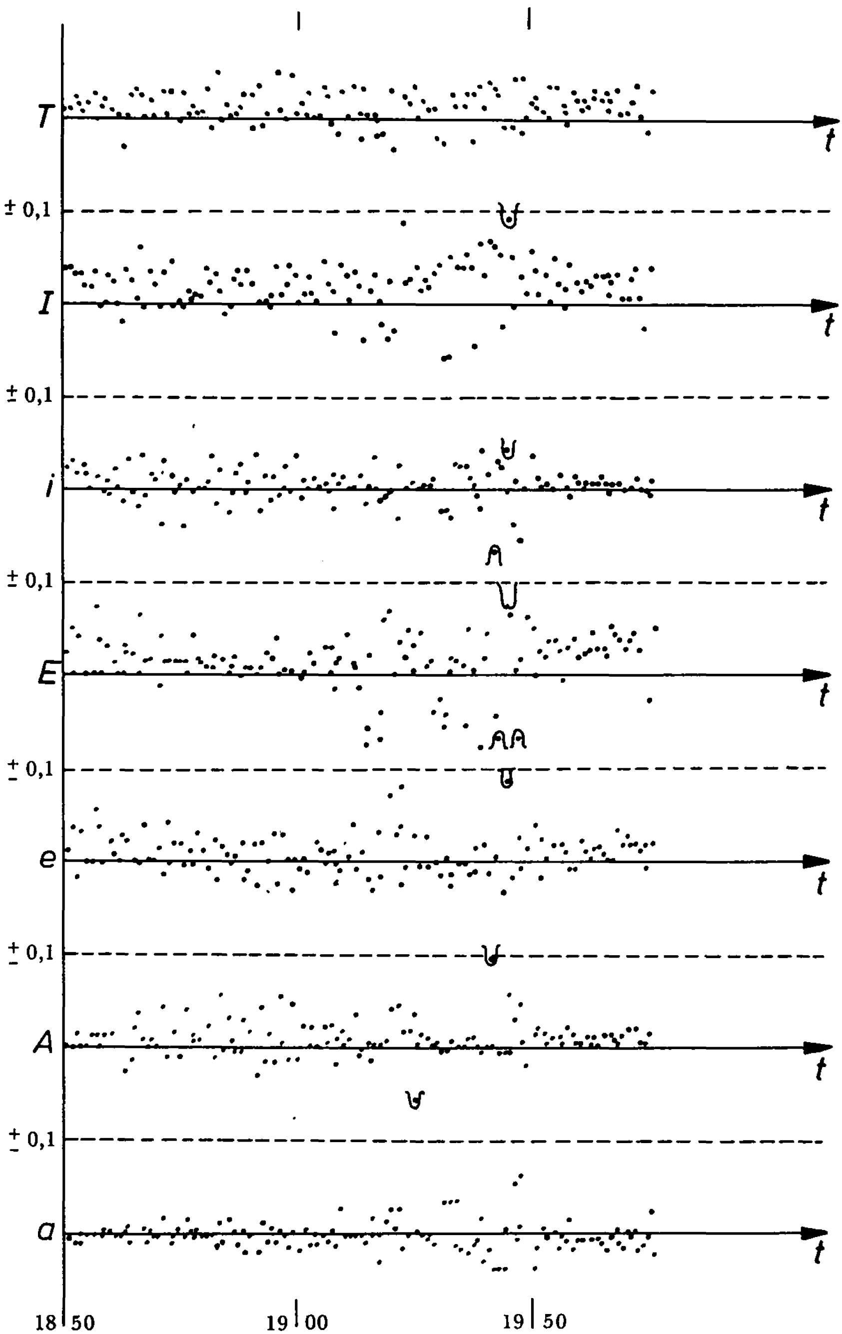

Table 1: Growth Rates of the Capitalist World Economy 1850–1976 (logarithmic scale 1 mm ≙ 0,0050/1 year ≙ 1 mm)

after World War II we can differentiate two periods in four of the seven series (in the others we cannot identify different periods). On the other hand, the six-dimensional cluster analysis shows us only one pre-war period and one post-war period, a fact which is caused by the shift of the single series.

But we have to ask ourselves whether the differences between the average growth rates are random or significant. As shown in formula (10a), these averages depend only on the last year of the given period (z_{in_i}) and the year before the first year of the given period ($z_{i0} = z_{i-1,\, n_{i-1}}$). Although the within-cluster differences are considered in formula (5), it is useful to test the differences between the average growth rates.

First, we test whether the data are normally distributed. By means of the Kuiper test[7] we prove that the logarithms of the indices are normally distributed, i. e. the indices are log-normally distributed. If the logarithmic indices are normally distributed, then the logarithmic growth rates

$$q_t = z_t - 2 \tag{11a}$$

are also normally distributed. The variance of the logarithmic growth rates is of the same magnitude as the variance of the logarithmic indices:

$$D^2(q) = D^2(z) \tag{11b}$$

Because the variances of q or z are different in different periods we test the significance of the differences between two average (logarithmic) growth rates by means of the approximate test by Welch[8]. The data needed for computing the test variable t and the degree of freedom f are just the same as for computing the objective function (5a). The average differ significantly only if

$$t = \frac{x_{ik} - x_{i-1,\,k}}{\sqrt{V^2(x_{ik})\, x^2_{ik}/n_i - V^2(x_{i-1,\,k})\, x^2_{i-1,\,k}/n_{i-1}}} > t_{f;\alpha} \tag{12a}$$

with

$$f = \frac{(n_i - 1)(n_{i-1} - 1)(V^2_{ik} x^2_{ik} n_{i-1} + V^2_{i-1,\,k} x^2_{i-1,\,k} n_i)^2}{V^4_{ik} x^4_{ik} n^2_{i-1}(n_{i-1} - 1) + V^4_{i-1,\,k} x^4_{i-1,\,k} n^2_i (n_i - 1)} \tag{12b}$$

and α as the significance level (95 %).

[7] Kuiper, N. H., Tests Concerning Random Points on a Circle, in: Proc. Konink. Ned. Akad. Wet., A 63 (1960), pp. 38 ff.

[8] Welch, B. L., The Significance of the Difference Between Two Means When the Population Variances are Unequal, in: Biometrica, 29 (1938), pp. 350 ff.

Because of the fact that we use an agglomerative method for the clustering of time series, we can not directly combine the Welsh test with this method. Enumerating the solutions of the objective functions c_k or C according to the number of clusters, we first have to compute the solutions

$c_k^{(N)}, c_k^{(N-1)}, \ldots, c_k^{(1)}, \ldots, c_k^{(2)}, c_k^{(1)}$. We then carry out the paired comparison of the averages $x_{ik}^{(1)}$ and $x_{i-1,k}^{(1)}$. The figure of this desagglomeration (or division) is the inverted dendrogramme which reflects the process of agglomeration. The results of cluster analysis and Welsh test are summarized in Appendix II.

We can say there have been periods of slower growth and periods of faster growth in respect to both industrial production and the ratio of industrial to total production. But this change is not statistically significant. If we assume a level of confidence probability of 95 %, then we find only one pair of logarithmic indices (or logarithmic growth rates) which differ significantly. All the other z_i differ significantly only at a lower level of confidence probability, that lies between 75 % and 90 %.

From a scientific standpoint we can only say that there have been differences between the cluster-means but the differences are too slight (or the variances of the means are too large) to be considered significant. Speaking positively, such a statement is an invitation for further research.

The results of clustering and testing export series are completely different. We are able to discriminate clearly three periods: 1850/66, 1867/1949, 1950/1976 the means of which differ significantly. The subperiods 1867/1893, 1894/1913 and 1913/1949 are very similar to each other. The growth of agricultureal production and of its ratio to total production is relatively regular: The cluster means differ very little (whereas the coefficients of variation are similar to those of the other series).

The answer to our original question is not very satisfying: There have been differences but most of them are not significant. The main reason for this is that variances within the clusters are very large. Most of the approximate coefficients of variation of the growth rates (i. e. $v(r_i)$) are greater than one hundred per cent. Moreover, we are able to prove that the size of $v(r_i)$ depends in such a way on r_i that the smaller the absolute value of r_i the larger its coefficient of variation. We construct a two-by-two table to show this fact. For that purpose we compare r_{ik} and $v(r_{ik})$ with r_k and $v(r_k)$ respectively. The table shows that high values of $v(r_{ik})$ are related to low values of r_{ik}:

	$r_{ik} > r_k$	$r_{ik} < r_k$
$v(r_{ik}) > v(r_k)$	1	9
$v(r_{ik}) < v(r_k)$	16	7

By means of Chi-squared-test (Yates correction included) we show that the probability of the mentioned dependence is more than 99 % (by means of the Fisher-Yates test we obtain a similar result: The rejection probability for our hypothesis is only 0.21 %)[9].

So we can conclude that the slower the growth of capitalist world economy the stronger the irregularities affecting the growth rates. Therefore it is not very useful to treat the time before World War I as a homogenous period. Quite clearly, there was a break in the development which relates to the transition of capitalist world economy to its imperialist stage. This break is visible in the series on industrial production and its ratio to total production. After the transition period, at the end of the last century, growth rates began to increase again. But that did not occur in the field of foreign trade. The growth rates of export volume increased slightly, but the international division of labor the ratio of total exports to total production hardly changed. Protectionism continued, and only after World War II did a strong upswing period set in.

Looking at the period after World War II it seems very probable that in the second half of the sixties a long down-swing period began. Too short a time has gone by to decide this question definitively but some indicators give support to such an opinion. So we may assume that the growth of the capitalist world economy over a longer period will be slower and more unstable than in the first twenty years after World War II.

[9] See e. g. Fisher, R. A., Statistical Methods for Research Workers, London 1948, p.95.

Figure 1: Industrial Production
(100 Mill. Dollars in constant prices of 1913)

Year	...0	...1	...2	...3	...4	...5	...6	...7	...8	...9
185.	31	34	37	40	42	44	48	47	47	50
186.	53	53	47	51	54	55	63	62	67	70
187.	68	74	82	81	81	79	82	83	85	87
188.	94	99	106	110	107	106	112	121	126	137
189.	144	146	148	143	148	162	166	174	190	204
190.	205	213	230	235	236	260	272	280	257	282
191.	302	307	334	350	319	329	354	357	336	308
192.	326	280	343	361	382	413	427	455	476	511
193.	448	389	333	375	417	466	525	578	518	606
194.	651	777	907	1033	980	679	578	651	700	714
195.	812	882	903	970	970	1085	1138	1176	1145	1274
196.	1358	1407	1500	1576	1705	1831	1975	2004	2130	2288
197.	2335	2382	2568	2802	2825	2638	2895			

Figure 2: Agricultural Production
(100 Mill. Dollars in constant prices of 1913)

Year	...0	...1	...2	...3	...4	...5	...6	...7	...8	...9
185.	116	116	120	120	120	124	128	128	132	136
186.	136	136	128	124	132	144	148	148	152	152
187.	168	164	172	172	168	184	184	192	196	200
188.	212	196	224	216	232	236	232	228	244	256
189.	236	264	252	260	252	288	292	280	312	300
190.	316	312	332	328	328	348	368	348	356	372
191.	372	376	408	400	388	392	352	352	352	360
192.	396	396	440	456	472	512	512	528	544	544
193.	548	544	540	548	548	548	564	592	588	576
194.	576	576	572	552	536	460	524	560	624	632
195.	596	612	640	664	668	688	708	708	740	764
196.	780	788	808	820	844	848	880	912	936	944
197.	964	996	996	1044	1060	1080	1118			

Figure 3: Total Exports
(100 Mill. Dollars in constant prices of 1913)

Year	...0	...1	...2	...3	...4	...5	...6	...7	...8	...9
185.	17	19	19	21	21	21	25	27	27	29
186.	30	30	30	32	34	36	42	42	44	46
187.	44	49	51	53	55	57	59	59	65	68
188.	70	72	76	78	82	82	84	89	89	91
189.	93	93	95	95	101	106	116	116	118	120
190.	118	120	120	127	131	141	152	154	146	152
191.	160	177	184	190	158	137	144	125	114	131
192.	152	152	165	173	194	207	209	232	238	247
193.	226	211	184	186	194	201	207	232	213	213
194.	174	183	202	262	232	158	188	220	224	236
195.	273	308	304	323	338	367	399	422	410	441
196.	485	509	538	578	636	685	736	769	875	970
197.	1054	1128	1233	1381	1465	1370	1539			

Figure 4: Total Production Including Mining
(100 Mill. Dollars in constant prices of 1913)

Year	...0	...1	...2	...3	...4	...5	...6	...7	...8	...9
185.	149	153	160	163	165	171	180	179	183	190
186.	193	193	179	180	191	204	216	216	225	228
187.	242	245	262	261	257	271	274	283	290	296
188.	316	306	342	339	351	354	357	363	385	408
189.	396	427	417	420	417	469	478	475	525	528
190.	547	552	590	593	594	642	677	667	649	694
191.	717	727	789	800	750	765	757	762	740	712
192.	773	716	832	874	911	986	1000	1050	1089	1130
193.	1065	990	921	973	1021	1076	1160	1250	1180	1259
194.	1310	1438	1566	1675	1604	1212	1182	1299	1419	1439
195.	1507	1602	1654	1748	1751	1897	1978	2021	2015	2175
196.	2284	2348	2469	2562	2725	2861	3028	3114	3275	3459
197.	3536	3618	3790	4112	4181	3996	4281			

Figure 5: Ratio Industrial to Total Production (Promille)
(100 Mill. Dollars in constant prices of 1913)

Year	...0	...1	...2	...3	...4	...5	...6	...7	...8	...9
185.	208	223	231	245	255	257	267	263	257	263
186.	275	275	263	283	283	270	292	287	298	307
187.	281	302	313	310	315	291	299	293	293	294
188.	297	324	310	324	305	299	314	333	327	336
189.	364	342	355	343	355	345	347	366	362	386
190.	375	386	390	396	397	405	402	418	396	406
191.	421	422	423	438	425	430	468	468	454	433
192.	422	391	412	413	419	419	427	433	437	452
193.	421	393	361	385	408	433	453	462	439	481
194.	497	540	579	617	611	560	489	501	493	496
195.	539	551	546	555	554	572	575	582	568	586
196.	595	599	608	615	626	640	646	664	650	661
197.	660	658	678	681	676	660	676			

Figure 6: Ratio Agricultural to Total Production (Promille)
(100 Mill. Dollars in constant prices of 1913)

Year	...0	...1	...2	...3	...4	...5	...6	...7	...8	...9
185.	779	758	750	736	727	725	711	715	721	716
186.	705	705	715	689	691	706	685	685	675	667
187.	694	669	656	659	654	679	671	678	676	676
188.	671	641	655	637	661	667	650	628	634	627
189.	596	618	604	619	604	614	611	590	594	568
190.	578	565	563	553	552	542	539	519	549	536
191.	519	517	517	500	517	512	465	451	476	505
192.	512	553	529	522	518	519	512	503	500	481
193.	515	550	586	563	537	509	486	474	498	458
194.	440	400	365	330	334	380	443	431	440	439
195.	395	382	387	380	382	363	358	350	367	351
196.	342	336	327	320	310	296	291	300	286	273
197.	273	275	263	254	254	270	261			

Figure 7: Ratio Total Exports to Total Production (promille)
(100 Mill. Dollars in constant prices of 1913)

Year	...0	...1	...2	...3	...4	...5	...6	...7	...8	...9
185.	114	124	119	129	127	123	139	151	148	153
186.	155	155	168	178	178	176	194	194	196	202
187.	182	200	195	203	214	210	215	208	224	230
188.	221	235	222	230	234	232	235	245	231	223
189.	235	218	228	226	242	226	243	244	225	227
190.	216	217	203	214	221	220	224	230	225	219
191.	223	244	233	238	211	179	190	164	154	184
192.	197	212	198	198	213	210	209	221	219	219
193.	212	213	200	191	190	187	178	186	181	169
194.	133	127	129	156	145	130	159	169	158	164
195.	181	192	184	185	193	193	202	209	209	203
196.	212	217	218	226	233	239	243	247	267	280
197.	298	312	325	336	350	343	359			

Appendix II: Results of Cluster Analysis and Welch Test

z_i – arithmetic mean of the logarithmic indices within the i-th cluster
$D(z_i)$ – standard deviation of z_i
$V(z_i)$ – coefficient of variation of z_i (in %)
r_i – geometric mean of the growth rates within the i-th cluster $= 10^{(z_i - 2)}$ (in %)
$v(r_i)$ – approximate coefficient of variation of r_i (in %) $= D(z_i)/(z_i - 2)$
α – confidence probability (in %); we used a sequence 60 %, 70 %, 75 %, 80 %, 85 %, 90 %, 95 %, 97,5 %, 99 %, 99,5 %, 99,95 %; if α is lower than 60 % we wrote 50 %.

In the lower cluster we combined all those clusters the means of which are lower than that of the whole time series (1850–1976), and in the upper cluster we combined the others.

1. Industrial Production

Cluster	z_i	$D(z_i)$	$V(z_i)$	r_i	$v(r_i)$	α
1850–1866	2.01924	0.02587	1.28 %	4.8 %	134 %	75 %
1867–1896	2.01403	0.01714	0.85 %	3.3 %	122 %	80 %
1897–1913	2.01906	0.02039	1.01 %	4.5 %	107 %	80 %
1914–1950	2.00988	0.05294	2.63 %	2.3 %	536 %	90 %
1951–1969	2.02368	0.01363	0.67 %	5.6 %	58 %	80 %
1970–1976	2.01459	0.02479	1.23 %	3.4 %	170 %	
1850–1976	2.01564	0.03294	1.63 %	3.4 %	211 %	
1850–1913	2.01671	0.02013	1.00 %	3.9 %	120 %	75 %
1914–1950	2.00988	0.05294	2.63 %	2.3 %	536 %	90 %
1951–1976	2.02123	0.01727	0.85 %	5.0 %	81 %	
upper cl.	2.02080	0.01970	0.97 %	4.9 %	95 %	90 %
lower cl.	2.01200	0.03942	1.96 %	2.8 %	328 %	

2. Agricultural Production

Cluster	z_i	$D(z_i)$	$V(z_i)$	r_i	$v(r_i)$	α
1850–1880	2.00873	0.01579	0.79 %	2.0 %	181 %	60 %
1881–1894	2.00536	0.02932	1.46 %	1.2 %	547 %	60 %
1895–1915	2.00914	0.02193	1.09 %	2.1 %	240 %	70 %
1916–1950	2.00520	0.02408	1.20 %	1.2 %	463 %	85 %
1951–1976	2.01051	0.00835	0.42 %	2.4 %	79 %	
1850–1976	2.00781	0.02001	1.00 %	1.8 %	256 %	
1850–1915	2.00814	0.02098	1.04 %	1.9 %	258 %	50 %
1916–1950	2.00520	0.02408	1.20 %	1.2 %	463 %	85 %
1951–1976	2.01051	0.00835	0.42 %	2.4 %	79 %	
upper cl.	2.00944	0.01566	0.78 %	2.2 %	166 %	80 %
lower cl.	2.00524	0.02537	1.27 %	1.2 %	484 %	

3. Total Exports

Cluster	z_i	$D(z_i)$	$V(z_i)$	r_i	$v(r_i)$	α
1850–1866	2.02455	0.02472	1.22 %	5.7 %	101 %	90 %
1867–1893	2.01313	0.01331	0.66 %	3.1 %	101 %	60 %
1894–1913	2.01506	0.01368	0.68 %	3.5 %	91 %	85 %
1914–1949	2.00261	0.05502	2.75 %	0.6 %	2108 %	99 %
1950–1976	2.03016	0.02032	1.00 %	7.1 %	67 %	
1850–1976	2.01553	0.03432	1.70 %	3.6 %	220 %	
1850–1913	2.01623	0.01567	0.78 %	3.8 %	97 %	90 %
1914–1949	2.00261	0.05502	2.75 %	0.6 %	2108 %	99 %
1950–1976	2.03016	0.02032	1.00 %	7.1 %	67 %	
upper cl.	2.02235	0.02226	1.10 %	5.3 %	100 %	99 %
lower cl.	2.00712	0.04255	2.12 %	1.7 %	598 %	

4. Total Production

Cluster	z_i	$D(z_i)$	$V(z_i)$	r_i	$v(r_i)$	α
1850–1866	2.01008	0.01483	0.74 %	2.3 %	147 %	60 %
1867–1893	2.00734	0.01687	0.84 %	1.7 %	230 %	90 %
1894–1913	2.01190	0.01978	0.98 %	2.8 %	166 %	85 %
1914–1950	2.00743	0.02650	1.32 %	1.7 %	357 %	97.5 %
1951–1966	2.01894	0.01293	0.64 %	4.5 %	68 %	75 %
1967–1976	2.01482	0.01657	0.82 %	3.5 %	112 %	
1850–1976	2.01050	0.01938	0.96 %	2.7 %	185 %	
1850–1913	2.00948	0.01728	0.86 %	2.2 %	182 %	
1914–1950	2.00743	0.02650	1.32 %	1.7 %	357 %	85 %
1951–1976	2.01736	0.01433	0.71 %	4.1 %	83 %	97.5 %
upper cl.	2.01465	0.01642	0.82 %	3.4 %	112 %	85 %
lower cl.	2.00825	0.04600	2.29 %	1.9 %	558 %	

5. Ratio Industrial to Total Production

Cluster	z_i	$D(z_i)$	$V(z_i)$	r_i	$v(r_i)$	α
1850–1869	2.00939	0.01653	0.82 %	2.2 %	176 %	95 %
1870–1896	2.00198	0.02055	1.03 %	0.5 %	1038 %	75 %
1897–1913	2.00590	0.01295	0.65 %	1.4 %	219 %	90 %
1914–1950	2.00245	0.02435	1.22 %	0.6 %	994 %	95 %
1951–1966	2.00493	0.00606	0.30 %	1.1 %	123 %	85 %
1967–1976	2.00197	0.00689	0.34 %	0.5 %	350 %	
1850–1976	2.00406	0.01826	0.91 %	0.9 %	450 %	
1850–1913	2.00512	0.01757	0.88 %	1.2 %	343 %	90 %
1914–1950	2.00245	0.02435	1.22 %	0.6 %	994 %	99.5 %
1951–1976	2.00739	0.00643	0.32 %	0.9 %	170 %	
upper cl.	2.00670	0.01276	0.64 %	1.6 %	190 %	90 %
lower cl.	2.00221	0.02119	1.06 %	0.5 %	959 %	

6. Ratio Agricultural to Total Production

Cluster	z_i	$D(z_i)$	$V(z_i)$	r_i	$v(r_i)$	α
1850–1880	1.99784	0.00801	0.40 %	–0.5 %	371 %	70 %
1881–1890	1.99486	0.01345	0.67 %	–1.2 %	262 %	60 %
1891–1915	1.99732	0.01127	0.56 %	–0.6 %	430 %	50 %
1916–1950	1.99679	0.02822	1.41 %	–0.7 %	879 %	80 %
1951–1966	1.99164	0.00987	0.50 %	–1.9 %	118 %	60 %
1967–1976	1.99436	0.01579	0.79 %	–1.3 %	280 %	
1850–1976	1.99614	0.01740	0.87 %	–0.9 %	453 %	
1850–1915	1.99720	0.01014	0.51 %	–0.6 %	362 %	50 %
1916–1950	1.99679	0.02822	1.41 %	–0.7 %	879 %	80 %
1951–1976	1.99268	0.01215	0.61 %	–1.7 %	166 %	
upper cl.	1.99329	0.00987	0.50 %	–1.5 %	147 %	90 %
lower cl.	1.99729	0.02396	1.20 %	–0.6 %	884 %	

7. Ratio Total Exports to Total Production

Cluster	z_i	$D(z_i)$	$V(z_i)$	r_i	$v(r_0)$	α
1850–1866	2.01447	0.02297	1.14 %	3.3 %	159 %	
1857–1893	2.00244	0.02085	1.04 %	0.8 %	855 %	95 %
1894–1913	2.00106	0.02107	1.05 %	0.1 %	1988 %	50 %
1914–1949	1.99553	0.03775	1.89 %	–0.7 %	845 %	75 %
1950–1976	2.01263	0.01371	0.68 %	2.7 %	109 %	99 %
1850–1976	2.00396	0.02676	1.34 %	0.9 %	676 %	
1850–1913	2.00506	0.02203	1.10 %	1.2 %	435 %	
1914–1949	1.99553	0.03775	1.89 %	–0.7 %	845 %	90 %
1950–1976	2.01263	0.01371	0.68 %	2.7 %	109 %	99 %
upper cl.	2.01331	0.01728	0.86 %	3.1 %	130 %	99 %
lower cl.	1.99911	0.02925	1.46 %	–0.2 %	3287 %	